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Relation of Speed of Centrifugal Pumps to Height of Lift

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RELATION OF SPEED OF CENTRIFUGAL PUMPS TO HEIGHT OF LIFT

BY

EDWARD JOHN MEHREN

THESIS

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This is to certify that the following thesis prepared under the direction of Professor A. N. Talbot, Head of the Department of Theoretical and Applied Mechanics, by

EDWARD JOHN MEHREN

entitled RELATION OF SPEED OF CENTRIFUGAL PUMPS TO HEIGHT OF LIFT

is hereby approved by me as fulfilling this part of the requirements for the Degree of Bachelor of Science in Civil Engineering.

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Relation of Speed of Centrifugal Pumps to Height of Lift.

Introduction.

The improvements that have been made in recent years in centrifugal pumps, have resulted in an increase in their use, and in new designs adapted to special purposes. Of the many improvements thus made none is more noteworthy than the multiple runner pump, which is now designed for heads of 500 feet or more, in one case* that has come to notice, for a head of 813 feet. But even in the more limited sphere of the single runner pump, study and experience have brought forth such adaptations and changes as to force the attention of engineers to the theoretical part of the subject. Another reason for the attention which has been so lavishly given of late to the theory of the centrifugal pump, is the necessity for such theory in order to design intelligently for the great variety of conditions under which such pumps are now called upon to operate.

* Built by the Byron Jackson Machine Works, San Fran.

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Despite the present wide use of centrifugal pumps, the theory has apparently made insignificant advance, when compared with the increase in use and the mechanical perfection of the pump. True it is that from the very nature of the subject, a theory that would have had the approbation of students of applied mechanics, might have been developed fifty years ago. That this was not done, and, what is more striking, that it has not yet been done, is evidenced by the discussion which arises on every presentation of the subject. The differences in the formulae developed are quite marked though the theories lately advanced happily show a convergence toward one set of formulae. The differences arise from the various assumptions made, and from a desire for simple formulae, with a consequent neglect of the lesser factors affecting the operation of the pumps.

Development of the Centrifugal Pump.

The method of raising water by means of some rotating device, is one of the earliest in the history of water pumping. These rotating devices

passed through various stages of development, and finally led to the only economical machine involving that principle, which has yet been devised, the centrifugal pump. In 1754 the eyes of the scientific world were first attracted to the centrifugal pump when Euler presented an account of his experiments in the Proceedings of the Academy of Berlin. Various experiments followed Euler's, but not until 1830 was there any device discovered to compare at all with the centrifugal pump of the present day; it was erected by Mc Carty in the New York navy-yard. In 1851 Mr. J.G. Appold brought out his centrifugal pump at the London Exhibition, and from that year, it can be truly said, dates the really important part of the history of the centrifugal pump. Appold found that the efficiency of the centrifugal pump depended mainly on the curvature of the blades; the blade curving backward giving the highest efficiency.

In 1876 Hon. R.C. Parsons presented a paper* before the Institution of Civil Engineers in which he discussed the experiments he had been making on circular and spiral pump casings. His results showed plainly the superiority

* Minutes of Proceedings, Inst. C.E., Vol. XLVII, p. 267

of the spiral casing. The paper was a valuable one, and such was its influence that Barr took from it all the theoretical matter pertaining to centrifugal pumps which is found in his "Pumping Machinery."

The Theory of the Centrifugal Pump.

The theory of the centrifugal pump has been approached from two widely differing standpoints. One school has endeavored to treat mathematically all phases of the question, not only the height of lift and the discharge, but also the losses; the other school applies mathematics only to the questions of height of lift as affected by the forces acting on the water, and treats the various losses either from experimental data, or by analogy to some similar mechanical contrivances. The one school tires by its unwarranted mathematical minuteness; the other, while inviting investigation by the simplicity of its formulae, destroys confidence because some of its investigators regard too lightly the theoretical aspect of some heavy sources of losses.

In studying the various theories, considerable difficulty was experienced because of the many different systems of notation in use. Every investigator uses a system of his own. In order, therefore, to make comparison easy, a standard system of notation (being that used by Mr. E. S. Harris, Mem. Am. Soc. C. E.*) was adopted, and all the theories are reduced to and stated in this notation throughout this discussion.

The following is the notation that will be used:

r = radius of outer limit of vanes

r_1 = " " inner " " "

x = " " any intermediate point - a variable

u = velocity of revolution of outer limit of vanes

u_1 = " " " " inner " " " $= \frac{r_1}{r} u$

V = Velocity of escaping water, relative to revolving wheel

V_1 = " " entering " " " " "

v = absolute velocity of escaping water

v_1 = " " " water as it enters vanes

W = weight of a cubic unit (foot) of the liquid

β = angle between V and u

θ = angle of discharge = angle between v and u

φ = angle between v_1 and u_1. $\varphi = 90°$, unless

* "Theory of Cent. Pumps & Fans", Trans. Am. Soc. C.E., Vol. LI, p. 166

guide vanes be inserted.

h = head or lift above pump

H = total effective lift, from lower water level to upper water level.

k = initial or entrance head, which is usually suction or negative head

p = pressure head in space between wheel and casing = pressure head in water at instant after escape from wheel.

w = radial velocity of water at exit from wheel

w_1 = " " " " " " entrance to " $= v_1$

t = velocity of water at end of discharge pipe

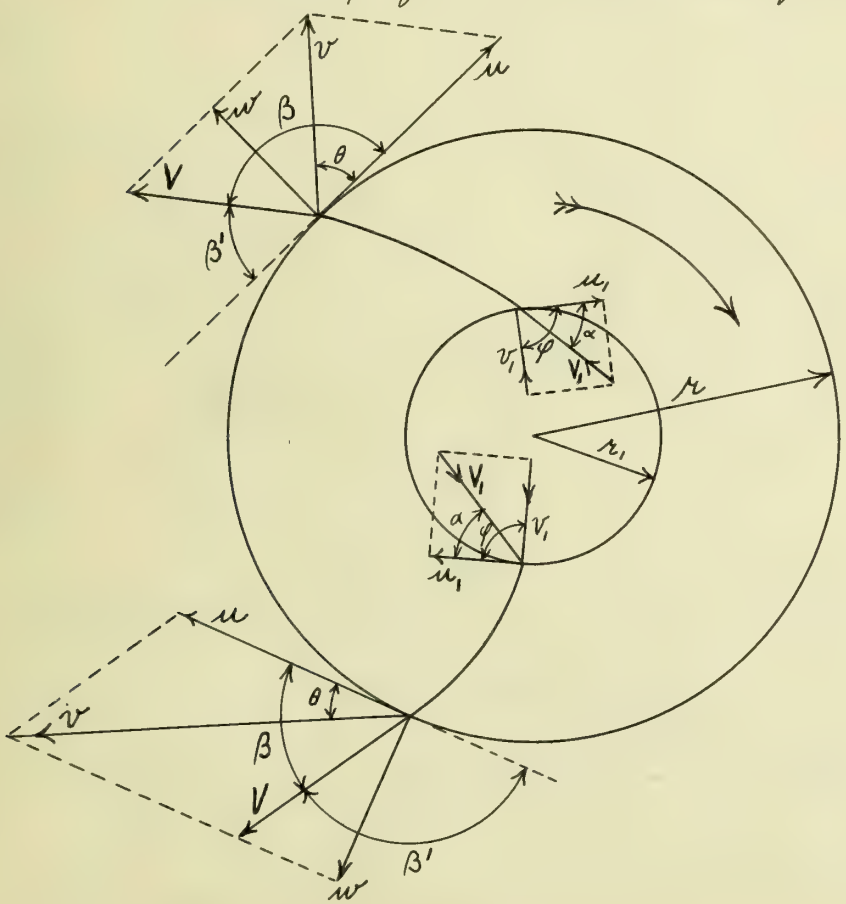

Fig. 1.

As mentioned above Parsons theory was one of the first in point of time. He advanced the idea that there were two forces acting on the water, centrifugal force and the "force of impact"; i.e. "the force with which the blades of the fan impinge against the water discharged by the pump."

If there be no discharge, the water in the pump will rotate with the vanes and centrifugal force will alone be active. The value of this force was thus deduced by Parsons:

The equation for centrifugal force is

$$F = \frac{m u^2}{r}$$

where u is the velocity of rotation at a distance r from the center of rotation.

$$\frac{m u^2}{r} = m r \omega^2$$

where ω = angular velocity

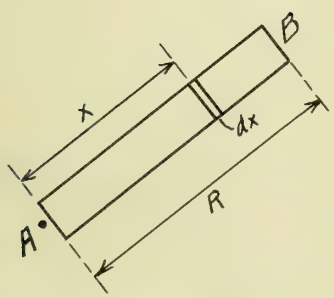

Fig. 2

Let AB be a prism of water rotating about A

The total centrifugal force $= \int_0^R m x \omega^2 dx$

$$= \int_0^R \frac{W}{g} \omega^2 x dx = \frac{W}{g} \omega^2 \frac{R^2}{2}$$

since $\omega = \frac{u}{R}$ total force $= W \frac{u^2}{2g} = F_1$

The force F_1 (neglecting friction) will support a pressure = Wh h = height of a column of liquid and W is unit weight

$$\text{or } F_1 = Wh$$
$$\therefore h = \frac{u^2}{2g} \qquad [1]$$

[It will be noted that if the lower limit in the integration be r_1 instead of 0, that the formula becomes

$$h = \frac{u^2 - u_1^2}{2g} \qquad [2]$$

This form is often met with.]

These formulae [1] and [2] correctly express the relation of the height of lift to the velocity of the runner when there is no discharge.

<u>A Fallacy.</u> – It may not be out of place to point out here the fallacy that has become rather widespread of considering

$$\frac{u^2}{2g} \quad \text{or} \quad \frac{u^2 - u_1^2}{2g}$$

as the maximum theoretical height of lift of centrifugal pumps. This is the maximum head that can be maintained when the pump is not discharging, and is due solely to centrifugal force; but as soon as the pump begins to

discharge another force comes into play, and the above expression no longer holds true. The fallacy has probably arisen from the failure to recognize that there is another factor in addition to centrifugal force.

Having determined the head due to centrifugal force when there is no discharge, Parsons proceeds to find the height of lift when there is a discharge, and finally derives an expression for the head due to impact. This last-mentioned expression gives the head $\underline{h'}$ due to the absolute tangential velocity of the water $\underline{u'}$ and is equal to

$$u - V \cos \beta' \qquad [3]$$

Barr

The theory advanced by Barr in his "Pumping Machinery" is copied from Parsons, but unfortunately has been so poorly copied, as to make it worse than worthless. Barr omits entirely the expression for the height of lift due to centrifugal force when there is a discharge (formula not given in this discussion), yet he copies Parsons' final formula without

explaining the change in Parsons' notation and without giving the value of $\underline{u}$ for the condition when there is a flow. In the expression

$$h + h' = \frac{u^2}{2g} + \frac{u'^2}{2g} \qquad [4]$$

Barr takes $\underline{h}$ to be the head due to centrifugal force when there is no discharge, whereas Parsons takes it as the $\underline{h}$ of his second expression (a very long one), which was derived for the condition of pump discharging.

Unwin

The next man to do notable work on the theory was Wm. C. Unwin, Mem. Inst. C. E. He endeavored to treat mathematically all the factors entering the operation of the pumps; and then applied his theories to the experimental data furnished by Parsons' paper. His discussion in the Proceedings of the Institution of Civil Engineers of 1878, Vol. LIII, remains today a noteworthy contribution to the study of centrifugal pumps.

He starts with the consideration of the various velocities involved, i.e. the velocity of the runner, and the absolute, relative and radial

velocities of the water, and the relations of these velocities at the entrance and discharge points of the vane. The variation of the pressure in the pump disk is next treated, and then in turn the influence of the inner and of the outer ends of the vanes. Equations are derived for the effects of the various forces acting in each case.

The various factors which have thus been separately considered, are then brought together and discussed for particular types of pumps. The general equation given for any centrifugal pump is this:

$$H = \frac{u^2 - u_1^2}{2g} - \frac{w^2 \operatorname{cosec}^2 \beta' - w_1^2 \operatorname{cosec}^2 \varphi}{2g}$$

$$+ \frac{u_1 w_1 \cot \varphi - w_1^2 \cot^2 \varphi}{g}$$

$$+ \frac{1}{g}\left\{ v_s (u - w \cot \beta') - v_s^2 \right\} - \frac{w_1^2}{2g} \quad [5]$$

v_s = velocity of water circulating around pump disk

The first term on the right is evidently the head due to centrifugal force. The second term is evidently equal to $\frac{V^2 - V_1^2}{2g}$

and is the head expended in producing the backward relative velocity of flow.

The third term on the right gives the gain in pressure (which goes to produce head) at the entrance to the vanes. When there are no guide vanes it becomes

$$\frac{w_1^2 - w_1'^2}{g} = 0 \quad \text{since } v_1 = w_1, \text{ and } w_1 \cot \varphi = u_1,$$

The fourth term is the gain in head due to the energy of the moving water at the periphery of the runner. In a poorly designed casing this becomes zero.

The fifth term is the velocity head at entrance to the vanes, and is generally negative in sign, thus increasing the head, H.

In the ordinary spiral casing pump the formula becomes

$$H = \frac{3}{4} \cdot \frac{u^2}{g} - \frac{2 u w \cot \beta' - 2V^2 - u^2 \cos^2 \beta'}{4g} \qquad [6]$$

If the gain of head due to the velocity of the wheel at the periphery of the runner be also neglected, the formula becomes

$$H = \frac{u^2 - V^2}{2g} \qquad [7]$$

It will be noted that this last expression

differs entirely from all the formulae which are studied in this paper. From the relation known to exist in actual tests between u and V, it is apparent that [7] will not give results at all approximating the results of experiments. This clearly shows then that Unwin's theory must be tested for the exact conditions for which its various factors are derived, and that any attempt to neglect them will result in nullifying the value of the formula.

Weisbach

Weisbach in his "Mechanics of Pumping Machinery" derives a formula which gives the value of H as

$$H = \frac{u^2}{g} \cdot \frac{\sin \beta' \cos \theta}{\sin (\beta' + \theta)} - \zeta - \frac{t^2}{2g} \quad [8]$$

ζ = all friction losses = friction loss in suction pipe, in discharge pipe, and through the wheel.

t = velocity at upper discharge level.

Hartmann - Knocke

Of the members of the school that treats mathematically in as far as possible all factors entering into the operation of the pump, Hartmann and Knocke are prominent. In "Die Pumpen" published in 1897 is found a very elaborate theory. The various stages in the passage of the water through the pump are studied separately as in Unwin's theory. The division made in their study is as follows: (1) Flow in suction pipe, (2) Entrance to the wheel, (3) Flow through the wheel, (4) Exit from wheel, (5) Flow in discharge pipe.

Their study is one of the best that has been examined and while following the general lines of Unwin's development, is far more satisfactory in its final result. The following formula is given for the effective lift.

$$H = \frac{u^2}{2g}\left(1 + \frac{\sin(\beta+\theta)}{\sin(\beta-\theta)}\right) - \zeta - \frac{t^2}{2g} \quad [9]$$

American Investigators

All of the American engineers who have devoted time to the theory of the centrifugal pump, have approached the subject with

the express idea of obtaining simple formulae. This they have succeeded in doing, but often at the cost of neglecting some of the important factors which influence the operation of the pump.

Merriman

Professor Merriman of Lehigh University in his "Treatise on Hydraulics" has a section on the centrifugal pump, in which he deduces in a very simple manner a formula for the relation of the speed to the height of lift. He assumes that the vanes run full of water.

If H_1 = pressure head at foot of disch. pipe
H_0 = " " top of suction pipe
h_0 = height of suction lift
h_1 = " " discharge "
h_a = " " water barometer
S = total head lost in friction

then

$$H_1 + \frac{V^2}{2g} - \frac{v^2}{2g} = H_0 + \frac{V_1^2}{2g} - \frac{v_1^2}{2g} \quad [10]$$

Neglecting friction

$$H_1 = h_a + h_1 - \frac{v^2}{2g} \quad \text{and} \quad H_0 = h_a - h_0 - \frac{v_1^2}{2g}$$

$$V_1^2 - V^2 - u_1^2 + u^2 - v_1^2 + v^2 = 2gH + \zeta \quad [11]$$

since $\varphi = 90°$ $\quad V_1^2 = u_1^2 + v_1^2$

and $\quad V^2 = u^2 + v^2 - 2uv\cos\theta$

we get by substituting these values of V_1 and V

$$H + \zeta = uv\cos\theta \quad [12]$$

Though this formula is unquestionably correct, Merriman could not have deduced it if he had followed his diagram and his argument. He states that the parallelograms of velocity are the same as in the reaction turbine, and in his development of the theory of the centrifugal pump he uses the same velocity parallelograms and the same notation as in the development of the theory of the reaction turbine. It is true that the method used in investigating the turbine is applicable to the investigation of the centrifugal pump; but the velocities at entrance do not bear the same relation to each other for the two cases. Merriman assumes that the relative velocity of the water at entrance to the vanes, V_1 is a component of the absolute velocity of the water, v_1. This is true in the reaction turbine, but in the centrifugal pump V_1 is the resultant of the two forces v_1 and u_1. The reason becomes evident on an inspection of the

figure on p. 6. That Merriman is wrong is further evidenced by the statement that the angle ϕ must equal zero in order that

$$V_1^2 = u_1^2 + v_1^2$$

ϕ must in fact be equal to 90°, and V_1 be the resultant of u_1 and v_1 in order that this may be true.

Harris

Of the later American investigators none has treated the subject more simply and yet more convincingly than Mr. E. G. Harris, M. Am. Soc. C. E., in a paper presented in 1903 before the Society and printed in the Transactions of the Society, Vol. LI. It is his notation that has been used in this paper. His paper aims to treat two classes of pumps, centrifugal and impulse. While maintaining that a purely impulse pump can be built, Mr. Harris concedes that a purely centrifugal pump is not possible along the present lines of construction.

Mr. Harris uses the same method for securing the expression

$$H = \frac{u^2 - u_1^2}{2g}$$

when there is no flow, as was used by Parsons (given above on p. 7.) The remainder of Harris' theory proceeds along different lines than followed by Parsons.

When there is flow, there will be an initial velocity V_1. The total head within the wheel at the circumference will be

$$\frac{V_1^2}{2g} + \frac{u^2 - u_1^2}{2g}$$

and since the velocity of flow relative to the wheel is V_1 we have

$$V = V_1^2 + u^2 - u_1^2 \qquad [13]$$

[It will be noted that Merriman's formula [10] would reduce to this, if the pressure heads at entrance and discharge are neglected.]

Formula [13] can also be derived by the following method:

Assume a particle moving outward through a radial tube, its velocity relative to the tube at any point being V_x

then $$\frac{u_x^2}{x} = \frac{d\,V_x}{dt} = \text{radial acceleration}$$

and $$V_x = \frac{dx}{dt} = \text{radial velocity}$$

then $$V_x \, dV_x = \frac{u_x^2 \, dx}{x}$$

but $$u_x = \frac{x}{r} u$$

$$\therefore \int_{V_1}^{V} V_x \, dV_x = \int_{r_1}^{r} \frac{u^2}{r^2} \cdot x \, dx$$

$$\frac{V^2 - V_1^2}{2} = \frac{u^2}{r^2} \cdot \frac{r^2 - r_1^2}{2}$$

$$\therefore V^2 - V_1^2 = u^2 - u_1^2 \quad [13]$$

According to Harris [13] holds whether the accumulation of centrifugal effect appears as pressure alone or as velocity alone, or partly in both; and holds for purely impulsive effects and for combined centrifugal and impulsive effects.

The absolute velocity of discharge is expressed by

$$v^2 = V^2 + u^2 + 2 u V \cos \beta \quad [14]$$

These formulae [13] and [14] hold only when $p = p_1$

p_1 = pressure head at entrance to vane where velocity is v_1

and p = pressure head at circumference of vane where velocity is v

Now taking these pressures into consideration and taking $\underline{k}$ as the total suction or entrance head we have

$$k = p_1 + \frac{v_1^2}{2g}$$

$$k + H = \frac{v^2}{2g} + p = h$$

where h = lift above pump

$$H = \frac{v^2}{2g} + p - k \qquad [15]$$

To eliminate p.

When no guide vanes are used

$$V_1^2 = v_1^2 + u_1^2$$

and [13] becomes

$$V^2 = v_1^2 + u^2$$

Then the head relative to the wheel within itself at the perimeter, due to centrifugal effects independent of pressures before entering is

$$\frac{v_1^2}{2g} + \frac{u^2}{2g}$$

If the total head at entrance is k, the pressure head there will be

$$k - \frac{v_1^2}{2g}$$

and the total head at the perimeter is

$$k - \frac{v_1^2}{2g} + \frac{v_1^2}{2g} + \frac{u_1^2}{2g} = k + \frac{u^2}{2g}$$

If the pressure at the point of escape is p then

$$V^2 = 2g\left(k + \frac{u^2}{2g} - p\right) = u^2 + 2g(k - p) \quad [16]$$

From [14], [15], and [16]

$$2gH = v^2 + 2g(p - k)$$

$$H = \frac{u^2 + uV\cos\beta}{g} \qquad [17]$$

Mr. Harris derives formula [17] by another very interesting method, and because of its novelty and the confidence it inspires in his method of approach, it is here given briefly.

Another method of deriving formula [17]: The work done by the wheel can be expressed in terms of the suction caused by the reaction of the escaping water on the wheel. This reaction which the wheel must overcome is $\frac{Wv}{g}$ and the component opposing the motion of the wheel is $\frac{Wv}{g}\cos\theta$. The work done in the suction is Wk, and the work done by the wheel in overcoming the reaction must be equal to $\frac{Wv}{g}\cos\theta \cdot u$. Now the water on being discharged must have a certain energy,

which, neglecting friction, must be equal to the work done by the wheel on the water. The kinetic energy of the water is $\frac{Wv^2}{2g}$ and the energy expended in discharge is Wp

Therefore $$Wk + \frac{Wv}{g} u \cos\theta = \frac{Wv^2}{2g} + pW = Wh \quad [a]$$

Hence $$2uv\cos\theta = v^2 + 2g(p-k)$$

Inserting the value of $2uv\cos\theta$ from the trigonometrical relation $V^2 = u^2 + v^2 - 2uv\cos\theta$

$$V^2 = u^2 + 2g(k-p) \qquad [16]$$

which is identical with equation [16]

Since $H = h - k$ (k being generally negative)

we have from [a]

$$H = \frac{v^2}{2g} + (p-k)$$

and with the aid of [14] we get

$$H = \frac{u^2 + uV\cos\beta}{g} \qquad [17]$$

identical with [17]

After finishing the development of his theory, Mr. Harris discusses present practice and from this discussion is quoted the following extract:

"One cause which adds to the confusion

in this matter [i.e. considering $\frac{u^2}{2g}$ as the theoretical lift] is that the efficiency of such pumps is not far from one-half, thus making the actual lift attained about one-half the theoretic lift, or

$$H = \frac{u^2 + uV\cos\beta}{2g}$$

instead of $H = \frac{u^2 + uV\cos\beta}{g}$ "

"It is very important that the pressure p in any test be measured, for by its aid we can interpret results much more intelligently than has been done in the past. For instance: The lift h above the pump should equal $\frac{u^2}{2g} + p$ and the deficiency shows what is lost after the water passes the joint, d [the periphery of the runner.] Thus we may determine where the greatest loss occurs."

Venable

In 1904 a paper on velocity pumps was presented before the division of Pumping Machinery of the International Engineering Congress at St. Louis, by Mr. W. M. Venable,

assoc. M. Am. Soc. C. E. (Printed in Trans. Am. Soc. C. E., Vol. LIV – Part D). As far as this investigation is concerned [the relation of lift to speed of pump] the paper is of little value. It is well to note, however, that Mr. Venable has quite an interesting treatment of the theory of the vane. Mr. Venable derives this formula,

$$g(H+S) = u^2 + u\,w\cot\beta \quad [8]$$

Mr. Venable also states that the maximum theoretic lift is $\frac{u^2}{g}$.

Goodenough

Some years ago Professor G. A. Goodenough of the University of Illinois developed a theory of the centrifugal pump which, while following the accepted method of integrating the various factors between the limiting values at entrance and at exit, departed from the beaten track in the preparatory analysis of these factors. His development has something akin to Weisbach's. He began with the forces at work, instead of with the velocities as is usually done; and the forces were determined by analyzing the accelerations of a particle traveling along a curve which has a

rotative motion. The development is one that would have been expected from a student of machine design, rather than from a civil engineer.

Goodenough's Theory.— A small mass of fluid moves from A to B along the tube AB in the wheel, which, meantime, is rotating. If f is the cross-section of the tube, and γ is the specific weight of the fluid, the mass of a length ds will be

$$m = \frac{\gamma f\, ds}{g}$$

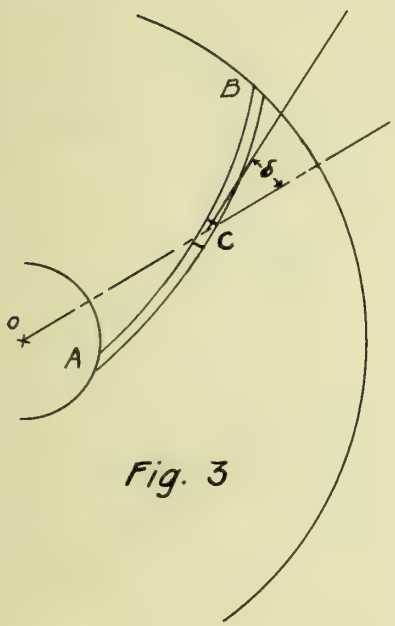

Fig. 3

Let the mass be at a point C at a distance x from the center. By Coriolis' law the acceleration of the mass at C has three components (1) the acceleration of the mass, assuming the path AB at rest, (2) the acceleration of the point of the wheel coincident with C, and (3) an acceleration normal to the tangent CT. These accelerations are resolved into their components in figure 4. If V is the velocity in the curve,

$\frac{dV}{dt}$ is the tangential acceleration, and $\frac{V^2}{\rho}$ the normal acceleration, ρ being the radius of curvature at C; these are the components mentioned under (1). The components of (2) are the centripetal acceleration $x\omega^2$, due to the angular velocity ω, and the tangential acceleration $x\frac{d\omega}{dt}$ which arises when ω is variable. The acceleration mentioned under (3) is normal to the path and has a value $2V\omega$.

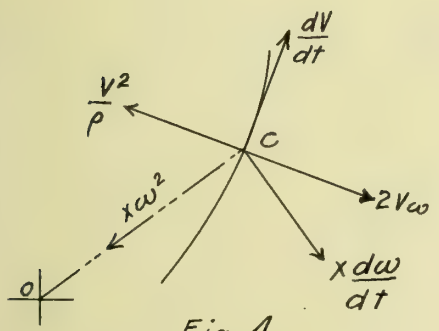

Fig. 4

The forces producing these accelerations are m times these accelerations. By D'Alembert's principle the system is equivalent to the system of external forces acting on the mass; i.e. the accelerating forces reversed, and the external forces are in equilibrium. The entire system of forces is shown in figure 5. The external forces are: (1) pressure dS between the small mass and the side of the tube; (2) the friction dF between the fluid and the tube; (3) a pressure p on the cross-section nearer

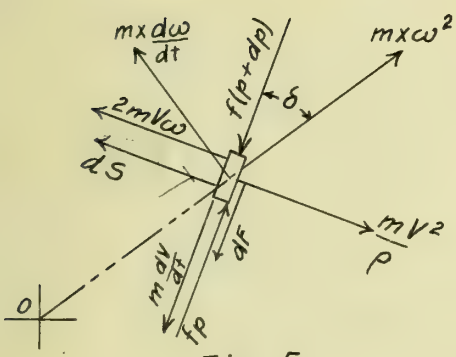

Fig. 5

the inner circumference. Since the pressure varies along the tube, the pressure at a distance ds from the point where the pressure is p f will be $f(p+dp)$. The nine forces of figure 5 are in equilibrium; hence resolving along the tangent to the path, we obtain

$$f(p+dp) - fp + m\frac{dV}{dt} - mx\omega^2\cos\delta + dF = 0 \quad [19]$$

$$\text{but } m = \frac{\gamma f\, ds}{g}$$

$$\text{then } f\cdot dp + \frac{\gamma f\, ds}{g}\cdot\frac{dV}{dt} - \frac{\gamma f x\omega^2 ds\cos\delta}{g} + dF = 0$$

$$\text{But } V = \frac{ds}{dt} \quad \text{and} \quad ds\cos\delta = dx$$

$$\text{then } \int_{p_1}^{p} dp + \gamma\int_{V_1}^{V}\frac{V\,dV}{g} - \frac{\gamma\omega^2}{g}\int_{r_1}^{r} x\,dx + \frac{1}{f}\int dF = 0$$

Integrating

$$[p-p_1] + \frac{\gamma}{2g}\left[V^2 - V_1^2\right] - \frac{\gamma\omega^2}{2g}\left[r^2 - r_1^2\right] + \frac{1}{f}\int dF = 0 \quad [20]$$

Since the form of function F is not definitely known we may denote $\frac{1}{f}\int dF$ by ζ_r and since $u_1 = r_1\omega$ and $u = r\omega$ we have

$$\frac{p-p_1}{\gamma} = \frac{1}{2g}\left[u^2 - u_1^2 + V^2 - V_1^2\right] - \zeta_r \quad [21]$$

Equation [21] may be interpreted: the change in

pressure head is equal to the change of velocity head less the friction head.

Now $V_1^2 = v_1^2 + u_1^2$ [22]

and $V^2 = v^2 + u^2 - 2uv\cos\theta$ [23]

and $\frac{V}{u} = \frac{\sin\beta}{\sin(\beta-\theta)}$ [24]

Substituting the above values of V_1 and V in [21] we have

$$\frac{p-p_1}{\gamma} = \frac{1}{2g}\left[v_1^2 - v^2 + 2uv\cos\theta\right] - \zeta_r \quad [25]$$

Eliminating v by means of equation [24] we have

$$\frac{p-p_1}{\gamma} = \frac{u^2}{2g}\left[\frac{2\sin\beta\cos\theta}{\sin(\beta-\theta)} - \frac{\sin^2\beta}{\sin^2(\beta-\theta)}\right] + \frac{v_1}{2g} - \zeta_r \quad [25a]$$

With a fixed angle θ there must be some angle β that will give a maximum change of pressure for a given value of u. From [25] $p-p_1$ will be a maximum when the quantity in the brackets is a maximum. By differentiating and solving for the maximum it is found that the maximum occurs when

$$\cos\theta = \frac{\sin\beta}{\sin(\beta-\theta)}$$

This relation is satisfied when $\beta = \theta + 90°$

Now at entrance

$$h_a + k = \frac{v_1^2}{2g} + \frac{p_1}{\gamma} + \zeta_1 \quad [26]$$

where ζ_1 = friction head at entrance.

At discharge

$$\frac{p_2}{\gamma} + \frac{v^2}{2g} = \frac{t^2}{2g} + h + h_a + \zeta_d \quad [27]$$

where ζ_d = friction head due to resistance in discharge pipe. ∴ Subtracting [26] from [27] and transposing

$$\frac{1}{\gamma}[p - p_1] = h - k + \frac{v_1^2 - v^2}{2g} + \frac{t^2}{2g} + \zeta_1 + \zeta_d \quad [28]$$

Comparing with [25] we obtain

$$\frac{v_r \cos\theta}{g} = h - k + \frac{t^2}{2g} + \zeta_1 + \zeta_d + \zeta_r \quad [29]$$

Eliminating v by means of [24], calling $H = h - k$, and $\zeta = \zeta_1 + \zeta_d + \zeta_r$ we get

$$\frac{u^2}{g} \cdot \frac{\sin\beta \cos\theta}{\sin(\beta - \theta)} = H + \frac{t^2}{2g} + \zeta \quad [30]$$

Professor Goodenough like Professor Merriman considers the absolute velocity of the water on entrance as the resultant of the velocity of the wheel and the relative velocity of the water to the vane. As stated before

the absolute velocity of the water is a component of the relative velocity.

Comparison of Results

Reduction to a common form.

In order to compare the various formulae for H which have been presented they will be reduced to a standard form.

Unwin.

The writer was unable to reduce Unwin's formulae to any simple form that would aid in comparing them with other formulae.

Weisbach

Formula [8]

$$H = \frac{u^2}{g} \cdot \frac{\sin \beta' \cos \theta}{\sin(\beta' + \theta)} - \zeta - \frac{t^2}{2g} \qquad [8]$$

can be written

$$H = \frac{u^2}{g} \cdot \frac{\sin \beta \cos \theta}{\sin(\beta - \theta)} - \zeta - \frac{t^2}{2g}$$

Since $\sin(\beta - \theta) = \sin \beta \cos \theta - \cos \beta \sin \theta$

we have $H = \frac{u^2}{g}\left[\frac{(\sin \beta \cos \theta - \cos \beta \sin \theta) + \cos \beta \sin \theta}{\sin(\beta - \theta)}\right] - \zeta - \frac{t^2}{2g}$

which reduces to

$$H = \frac{u^2}{g}\left[1 + \frac{\cos\beta \sin\theta}{\sin(\beta-\theta)}\right] - \zeta - \frac{t^2}{2g} \quad [31]$$

$$\text{or } u = \sqrt{g\left(H+\zeta+\frac{t^2}{2g}\right)\left(\frac{\sin(\beta-\theta)}{\sin(\beta-\theta)+\cos\beta\sin\theta}\right)}$$

$$u = \sqrt{g\left(H+\zeta+\frac{t^2}{2g}\right)(1-\cot\beta\tan\theta)} \quad [32]$$

Hartmann - Knoche

Formula [9]

$$H = \frac{u^2}{2g}\left(1 + \frac{\sin(\beta+\theta)}{\sin(\beta-\theta)}\right) - \zeta - \frac{t^2}{2g} \quad [9]$$

can be reduced to the same form as Weisbach's by manipulating the quantity in parenthesis

$$\left(1 + \frac{\sin(\beta+\theta)}{\sin(\beta-\theta)}\right) = 2\left(\frac{\sin\beta\cos\theta}{\sin(\beta-\theta)}\right) =$$

$$2\left(\frac{\sin\beta\cos\theta - \cos\beta\sin\theta + \cos\beta\sin\theta}{\sin(\beta-\theta)}\right) =$$

$$2\left(\frac{\sin(\beta-\theta) + \cos\beta\sin\theta}{\sin(\beta-\theta)}\right) = 2\left(1 + \frac{\cos\beta\sin\theta}{\sin(\beta-\theta)}\right)$$

Formula [9] therefore becomes

$$H = \frac{u^2}{g}\left(1 + \frac{\cos\beta\sin\theta}{\sin(\beta-\theta)}\right) - \zeta - \frac{t^2}{2g} \quad [31]$$

and $$u = \sqrt{g\left(H+\zeta+\frac{t^2}{2g}\right)(1-\cot\beta\tan\theta)} \quad [32]$$

Merriman

Formula [12]

$$H + \zeta = u\, v \cos\theta \qquad [12]$$

can be reduced to another form by taking the value of $u\,v\cos\theta$ from

$$V^2 = u^2 + v^2 - 2uv\cos\theta$$

and eliminating v by the equation

$$v^2 = V^2 + u^2 + 2uV\cos\beta$$

we then get

$$H = \frac{u^2 + uV\cos\beta}{g} - \zeta$$

or

$$H = \frac{u^2}{g}\left(1 + \frac{V}{u}\cos\beta\right) - \zeta \qquad [33]$$

but

$$\frac{V}{u} = \frac{\sin\theta}{\sin\{180^\circ - (\beta + \theta)\}} = \frac{\sin\theta}{\sin(\beta - \theta)}$$

$$\therefore \left(1 + \frac{V}{u}\cos\beta\right) = \left(1 + \frac{\cos\beta\sin\theta}{\sin(\beta-\theta)}\right)$$

$$\therefore H = \frac{u^2}{g}\left(1 + \frac{\cos\beta\sin\theta}{\sin(\beta-\theta)}\right) - \zeta \qquad [34]$$

Except for the omission of $\frac{t^2}{2g}$ [34] is identical with [31]

and $$u = \sqrt{g(H+\zeta)(1 - \cot\beta\tan\theta)} \qquad [35]$$

Harris

Formula [17]

$$H = \frac{u^2 + u V \cos \beta}{g} \qquad [17]$$

or

$$H = \frac{u^2}{g}\left(1 + \frac{V}{u} \cos \beta\right) \qquad [17a]$$

can be reduced as shown on the preceding page to

$$H = \frac{u^2}{g}\left(1 + \frac{\cos \beta \sin \theta}{\sin(\beta - \theta)}\right) \qquad [36]$$

Except for the omission of $\frac{t^2}{2g}$ and ζ [36] is identical with [31]

and $u = \sqrt{gH(1 - \cot \beta \tan \theta)}$ [37]

Venable

Formula [18]

$$g(H + \zeta) = u^2 + u w \cot \beta \qquad [18]$$

can be written

$$H = \frac{u^2 + u V \cos \beta}{g} - \zeta \qquad \text{since } w \cot \beta = V \cos \beta$$

or $$H = \frac{u^2}{g}\left(1 + \frac{V}{u} \cos \beta\right) - \zeta \qquad [33]$$

which as shown on page 32 is reducible to

$$H = \frac{u^2}{g}\left(1 + \frac{\cos\beta \sin\theta}{\sin(\beta-\theta)}\right) - \varsigma \qquad [34]$$

and $$u = \sqrt{g(H+\varsigma)(1-\cot\beta\tan\theta)} \qquad [35]$$

Goodenough

Equation [30]

$$\frac{u^2}{g} \cdot \frac{\sin\beta\cos\theta}{\sin(\beta-\theta)} = H + \frac{t^2}{2g} + \varsigma \qquad [30]$$

can be reduced to

$$H = \frac{u^2}{g}\left[1 + \frac{\cos\beta\sin\theta}{\sin(\beta-\theta)}\right] - \varsigma - \frac{t^2}{2g} \qquad [31]$$

as shown on pages

and $$u = \sqrt{g\left(H + \varsigma + \frac{t^2}{2g}\right)(1-\cot\beta\tan\theta)} \qquad [32]$$

Choice of a Standard Form.

The question of choosing a standard form for the equation expressing the relation of the speed to the lift, is an important one. Had a standard form been adopted years ago, much of the discussion on the theory of the centrifugal pump may never have occurred, and valuable time have been saved. It is striking how many different forms are used for the final expression, as many almost as there are investigators. For many reasons a form for the head which should involve the expression $\frac{u^2}{g}$ times an expression of the form of $(1 + \cdots)$ was deemed best; the principal reason being that u is the principal variable in any expression for the head. Each of the following forms has some advantages:

$$H = \frac{u^2}{g}\left(1 + \frac{v}{u}\cos\beta\right) \qquad [a]$$

$$H = \frac{u^2}{g}\left(1 + \frac{w}{u}\cot\beta\right) \qquad [b]$$

$$\text{and} \quad H = \frac{u^2}{g}\left(1 + \frac{\cos\beta\sin\theta}{\sin(\beta - \theta)}\right) \qquad [c]$$

For any given pump the variables in [a] would be the velocity of the runner, u, and

the relative velocity of the water to the wheel V; in [b], $\underline{u}$, and the radial velocity of the water on leaving the wheel, $\underline{w}$; in [c], $\underline{u}$, and the angle $\underline{\theta}$ between $\underline{u}$ and the absolute velocity of the water $\underline{v}$. In any of the three cases there would be two variables.

[a] and [b] are simpler of numerical application than [c], but [c] has the merit of not involving $\underline{u}$ within the parenthesis. It must be remembered, however, that the second term within the parenthesis is, in every case, a variable function of $\underline{u}$, and thus in fact the effect of $\underline{u}$ cannot be studied solely by a consideration of $\frac{u^2}{g}$ even in [c].

The numerical values of V, w, and θ, are determined with the aid of the following formulae:

$$V^2 = v_1^2 + u^2$$

$$w = V \sin \beta$$

$$v^2 = V^2 + u^2 + 2uV\cos\beta$$

and $$\sin\theta = \frac{V}{v}\sin\beta$$

Results Reduced to Standard Forms

The formulae obtained by the various investigators, reduced to the standard form, are here brought together for easy comparison. The value of $\underline{u}$ is also tabulated for ready reference. It should be remembered that any of the three forms (a), (b) or (c) can be used, by substituting the proper parenthesis

$$\left[1+\frac{v}{u}\cos\beta\right]=\left[1+\frac{w}{u}\cot\beta\right]=\left[1+\frac{\cos\beta\sin\theta}{\sin(\beta-\theta)}\right]$$

TABLE I

Investigator	Value of H	Value of $\underline{u}$
Unwin	Could not reduce to standard forms	
Weisbach	$\frac{u^2}{g}\left[1+\frac{v}{u}\cos\beta\right]-\zeta-\frac{t^2}{2g}$ [31]	$\sqrt{g(H+\zeta+\frac{t^2}{2g})(1-\cot\beta\tan\theta)}$ [32]
Hartmann-Knocke	$\frac{u^2}{g}\left[1+\frac{v}{u}\cos\beta\right]-\zeta-\frac{t^2}{2g}$ [31]	$\sqrt{g(H+\zeta+\frac{t^2}{2g})(1-\cot\beta\tan\theta)}$ [32]
Merriman	$\frac{u^2}{g}[\quad]-\zeta$ [34]	$\sqrt{g(H+\zeta)(1-\cot\beta\tan\theta)}$ [35]
Harris	$\frac{u^2}{g}[\quad]$ [36]	$\sqrt{gH(\quad)}$ [37]
Venable	$\frac{u^2}{g}[\quad]-\zeta$ [34]	$\sqrt{g(H+\zeta)(\quad)}$ [35]
Goodenough	$\frac{u^2}{g}[\quad]-\zeta-\frac{t^2}{2g}$ [31]	$\sqrt{g(H+\zeta+\frac{t^2}{2g})(\quad)}$ [32]

In the light of the discussion in recent years on centrifugal pumps in the American Society of Civil Engineers and by Hartmann-Knocke, the above correspondence of formulae is truly remarkable. In the introduction to their theory of the centrifugal pump in "Die Pumpen" (1897), Hartmann-Knocke state that there is great diversity in the conclusions reached in the various theories of the centrifugal pump, and that these different conclusions result from the variety of assumptions made, the simplifications used, and the neglect of some of the factors which enter the problem. Undoubtedly these formulae can differ in the values of ζ, for this depends on experiment, and the fact that Hartmann-Knocke themselves treat this factor ζ empirically, shows that their reference is to the theory and not to the friction losses.

M. E. G. Harris, M. Am. Soc. C.E., in the paper referred to above, said in his introduction, "The writer recently had occasion to look for a satisfactory discussion of the theory of centrifugal pumps ---- and was surprised to find that, apparently, the sub-

ject ---- had been overlooked or neglected by competent writers on applied mechanics."

Then, too, the general attitude of the profession toward any theory advanced has been one of suspicion, and each student of the subject is unwilling to stand upon the shoulders of those preceding him, but insists upon using some original method in his investigation. The formulae tabulated above, which are no more than simplifications of those derived by these investigators, show them to have secured identical results; but this fact has been so buried under the variety of mathematical forms, and in confusing and varying notation, that it has never before been brought to light.

Experimental Data.

While there has been, as shown above, ample discussion of the theory of the relation between the speed of the pump and the height of lift, there have been few, if any, experiments made to test this relation. There are tests without number to determine the efficiency

of pumps, but in all the cases that have been found, the pump has been run at the rated speed, or with a speed variation of only a few percent. These experiments are manifestly inadequate for testing thoroughly the relation of the speed to the height of lift. For this reason a series of experiments were devised by Professor A. N. Talbot of the University of Illinois, and a test along the lines laid down by him, was made by the writer, and is given herewith as a means of illustrating the method used. The tests were devised, too, with the idea of furnishing the pump manufacturer with tables and formulae which would aid him in the design of his pumps. The purchaser specifies the discharge and the head against which the pump is to work; and, therefore, it was desired to construct such tables and derive such formulae as would be helpful in determining the speed of the pump when these two factors were given. That there is a demand for such tables and formulae was evidenced when the now famous pumps for the New Orleans sewerage system failed to come up to specifications.

The Test

The pump tested is a 6 inch Goulds centrifugal pump, having a 24 inch runner with back-ward curved vanes. It was driven by an Ideal engine of 12 inch stroke, and 13 inch piston. A gate valve was placed on the suction pipe and the pump primed by running water into it through the discharge pipe.

Water was drawn from a sump and discharged through a line of 12 inch or a line of 8 inch pipe into a large weir chamber. The quantity of water discharged was measured over a 3 foot weir with suppressed end contractions. The coefficient of the weir was carefully worked out some years ago, and this coefficient was used in working up the data of this test. Gauges were placed on the suction and the discharge pipes, and the distances from the gauges to the center of the pump were measured.

The head pumped against, H, the effective lift, in the above theory, is made up of the reading of the suction gauge plus the distance from the center of the suction gauge to the center

of the pump, plus the distance from the center of the pump to the center of the discharge gauge, plus the head on the discharge gauge. The resistance through the pump, and the resistance and velocity in the discharge pipe are not included in the effective head, H.

The efficiency of the pump was calculated by the formula

$$\frac{\text{Hydraulic H.P}}{\text{H.P. delivered to pump runner}}$$

The hydraulic H.P. is found from the weight of the water pumped and the head pumped against, H. The H.P. delivered to pump runner is found by deducting the friction H.P. of the engine and the pump (the latter running empty), from the H.P. developed by the engine. The engine H.P. was determined from indicator cards. The speed of the engine was taken by means of a revolution counter and a watch, and the speed of the pump calculated from the speed of the engine by means of the ratio of the diameters of the two pulleys.

Three complete sets of readings were made for every condition, and one of these three

sets worked up for the accompanying data sheet. This large number of readings and cards was taken in order to be certain that the conditions of head, speed, and discharge were constant when the readings were taken. The weir chamber was quite large, and considerable time was required for the head on the weir to become constant, after a change of discharge head or of speed.

The range of speed of the pump and the range of the discharge are not as wide as was desired, but great difficulty was experienced with the belt. It seemed impossible to get it into shape so as not to slip. In fact two belts were more or less seriously damaged by running off the pulley.

On beginning the test the engine was run as slowly as possible and a reading taken with the discharge valve wide open. This gave the lowest head pumped against, and it could not be maintained if the valve were to be closed even by a slight amount. Three sets of readings were taken for this condition. The speed of the engine was then increased by four or five revolutions per min-

ute, a head on the discharge gauge determined, and the valve in the discharge pipe closed as much as possible without raising above the desired head. The accustomed three sets of readings were taken for this condition. The speed of the engine was again increased, the head being kept constant by open the valve in the discharge pipe, and three more sets of readings taken. This was continued until the discharge pipe was wide open. A new head was then determined upon, the discharge valve almost closed, and the engine speed regulated so as to maintain just this desired head. The test was then continued as above described. The heads were varied by five feet for the lower and by ten feet for the higher heads.

The only difference between this test and the one made on the same pump in 1903, is in the manner of measuring the suction; in 1903 it was measured directly, in this test by a vacuum gauge.

On May 4 the water was discharged through a 12 inch pipe, on April 27 and 28 through an 8 inch pipe.

TABLE 2
TEST OF 6 INCH GOULDS PUMP

Reference No.	Test No.	LIFT IN FEET			WATER PUMPED Gal. p'r Min.	R.P.M. of PUMP	H.P.		EFFICIENCY
		Discharge	Suction	Total			Deliv. to Pump	Hydraulic	
	Apr. 27								
1	6	8.5	4.3	12.8	650	318	2.96	2.10	.71
2	11	13.8	3.9	17.7	325	330	3.05	1.46	.48
3	15	14.3	4.3	18.6	575	345	3.99	2.71	.68
4	17	13.3	4.0	17.3	950	361	6.74	4.15	.62
5	21	14.1	4.3	18.4	575	349	4.51	2.69	.60
6	24	14.3	4.2	18.5	1000	384	8.16	4.69	.58
7	27	17.0	4.5	21.5	125	357	3.30	0.68	.22
8	30	20.3	3.8	24.1	500	392	7.00	3.05	.44
9	33	18.7	4.2	22.9	810	400	8.33	4.69	.56
10	39	24.5	3.8	28.3	300	423	8.25	2.15	.26
11	43	26.0	4.4	30.4	1025	465	16.0	7.89	.49
	Apr. 28								
12	2&3	24.0	3.4	27.4	190	404	6.29	1.32	.21
13	6	24.2	3.4	27.6	375	423	8.09	2.62	.32
14	9	23.6	3.9	27.5	780	435	11.58	5.43	.47
15	12	23.5	4.5	28.0	1025	454	14.43	7.26	.50
16	15	23.9	5.1	29.0	1300	481	19.23	9.54	.50
17	17	33.7	3.4	37.1	390	481	13.23	3.66	.28
18	21	32.8	3.6	36.4	650	489	15.62	5.99	.38
19	24	32.7	5.2	37.9	1325	531	25.73	12.7	.50
	May 4								
20	3	33.5	2.9	36.4	580	485	15.31	5.34	.35
21	6	40.8	2.8	43.6	225	508	14.61	2.48	.17
22	9	46.5	2.9	49.4	500	559	22.03	6.25	.28
23	11	45.5	3.2	48.7	825	563	27.13	10.2	.38
24	18	56.5	2.5	59.0	360	605	27.93	5.37	.19
25	20	51.5	2.6	54.1	525	590	26.63	7.20	.27

Other Tests of Goulds Pumps

This pump was tested in 1903 under the direction of Professor Talbot at the University of Illinois, and the data of that test show a much larger range both of discharge and speed. Since the two tests compare quite well, the data of the earlier test will be used.

This pump (6 inch discharge) was only one of a number of Goulds pumps which were tested by the above described method at the University of Illinois. The other tests were on pumps of the following sizes of discharge, $1\frac{1}{2}$, $1\frac{3}{4}$, 2, $2\frac{1}{2}$, 3, 4, 5 and 8 inches.

From the test data of these pumps, Professor Talbot derived formulae for the relation of the speed to the lift. These two forms were used:

$n = K\sqrt{h+a}$ (a) and $n = K_1\sqrt{g \times \text{total head}}$ (b)

where n = r. p. m. and K, K_1, and a are constants. The following method is that devised and used by Professor Talbot in deriving the constants for the above formulae.

Determination of Constants for the Above Formulae:

The value of a under the radical varied in the same pump, for the different discharges, and was determined by a series of computations, aided by some empirical work.

It was made up as follows:

1. Velocity head $\left(\frac{v^2}{2g}\right)$ in suction pipe, i.e. the head to produce the velocity in the suction pipe.

2. Calculated entrance head into suction pipe (using $.93\,\frac{v^2}{2g}$) for pipe projecting into water.

3. Calculated friction head in suction pipe, using the ordinary friction formula.

The sum of 1, 2, and 3, should equal the difference between the vacuum gauge reading and the distance from water level in the sump to center of gauge.

4. Velocity head $\left(\frac{v^2}{2g}\right)$ in discharge pipe, i.e. the head to produce velocity, $\underline{v}$, in discharge pipe.

The constant $\underline{a}$ was determined by adding to the sum of 1, 2, 3 and 4, the estimated loss through the pump. This last quantity was considered as a function of the velocity. The values of $\underline{h+a}$ and $\underline{n}$ were then platted on logarithmic co-ordinate paper, and the value of $\underline{K}$ determined for the particular pump under discussion. $\underline{K}$ for any given pump is the same for all rates of discharge, but is different

for different pumps. In determining $\underline{K}$, the more accurate value of $\underline{a}$ to make the curve fit was also determined.

Using the values of $\underline{h+a}$ as the total lift, the value of K_1 was determined. K_1 is constant for any pump, but is different for different pumps.

Table 3 shows in tabular form the formulae obtained for $\underline{n}$ for different pumps, and table 4 gives the value of $\underline{n}$ in terms of the total lift. In the latter table two forms are used

$$n = K_1 \sqrt{g H_T}$$

and $$H_T = \frac{n^2}{K_1^2 g}$$

H_T = total head = effective head + loss in suction + loss in discharge + loss through pump + velocity at discharge end of pipe = $S + \frac{t^2}{2g}$

TABLE 3

R.P.M. IN TERMS OF h FOR DIFFERENT PUMPS AND DISCHARGE

GALLONS PER MIN.	$1\frac{1}{2}$ IN.	$1\frac{3}{4}$ IN.	2 IN.	$2\frac{1}{2}$ IN.	3 IN.	4 IN.	5 IN.	6 IN.	8 IN.
50		$140\sqrt{h+1\frac{1}{2}}$	$112\sqrt{h+1}$						
90	$200\sqrt{h+10}$	$140\sqrt{h+6}$	$112\sqrt{h+3}$	$98\sqrt{h+5}$					
125	$200\sqrt{h+20}$	$140\sqrt{h+14}$	$112\sqrt{h+5}$						
180		$140\sqrt{h+27}$	$112\sqrt{h+12}$	$98\sqrt{h+10}$					
250				$98\sqrt{h+15}$					
300				$98\sqrt{h+20}$	$95\sqrt{h+4\frac{1}{2}}$	$95\sqrt{h+2}$			
400					$95\sqrt{h+10}$	$95\sqrt{h+4}$			
500					$95\sqrt{h+18}$	$95\sqrt{h+6\frac{1}{2}}$	$87\sqrt{h+2\frac{1}{2}}$		
600					$95\sqrt{h+30}$	$95\sqrt{h+10}$	$87\sqrt{h+4}$		
700						$95\sqrt{h+14}$	$87\sqrt{h+6}$		
800						$95\sqrt{h+19}$	$87\sqrt{h+8\frac{1}{2}}$	$75\sqrt{h+2\frac{1}{2}}$	
1000						$95\sqrt{h+31\frac{1}{2}}$	$87\sqrt{h+15}$	$75\sqrt{h+5}$	
1200						$95\sqrt{h+48}$	$87\sqrt{h+22\frac{1}{2}}$	$75\sqrt{h+7\frac{1}{2}}$	
1400								$75\sqrt{h+10}$	
1600								$75\sqrt{h+13}$	
1800								$75\sqrt{h+18}$	
2000								$75\sqrt{h+24}$	$80\sqrt{h+7\frac{1}{2}}$
2200								$75\sqrt{h+30}$	$80\sqrt{h+9}$
2500									$80\sqrt{h+11\frac{1}{2}}$
3000									$80\sqrt{h+16\frac{1}{2}}$

TABLE 4.
VALUES FOR GOULDS PUMPS

SIZE OF PUMP	VALUE OF $\underline{u}$	VALUE OF H_T
$1\frac{1}{2}$	$1.38\sqrt{g H_T}$	$.52 \frac{u^2}{g}$
$1\frac{3}{4}$	1.30	.59
2	1.30	.59
$2\frac{1}{2}$	1.13	.78
3	1.10	.83
4	1.17	.73
5	1.33	.56
6	1.37	.53
8	1.48	.46

Theoretical and Experimental Results Compared

On comparing the formula

$$H_T = \frac{u^2}{g}\left(1 + \frac{v}{u}\cos\beta\right) \qquad [31]$$

with the above values of H_T obtained experimentally, it becomes quite evident that the formula does not give the same results as the experiments. The expression $(1 + \frac{v}{u}\cos\beta)$ is a variable

whereas the above experiments show that it has a constant value for any particular pump. In order that $(1 + \frac{v}{u} \cos \beta)$ may be a constant, $\frac{v}{u}$ must be a constant. But v varies as the discharge, and since it is an indisputable fact that the discharge increases rapidly for a small increase in the speed of the pump, $\frac{v}{u}$ cannot be a constant. It would be interesting to know what factor should be introduced into [31] to harmonize it with practical results. Whether this new factor should modify the second term in the parenthesis or should be introduced as a third term is another point which needs to be determined.

It is interesting to note that Unwin in the paper above referred to (written in 1878) derived theoretically the following values of u for different values of β. w was taken as $\frac{1}{4}\sqrt{2gH} = .354\sqrt{gH}$

β	u
90°	$1.17\sqrt{gH}$
135°	1.31
150°	1.39
165°	$1.70\sqrt{gH}$

Conclusions

The study has brought out these facts:

1. That $\frac{u^2}{2g}$ is not the maximum theoretical height of lift, but is the maximum theoretical head that the pump can maintain when there is no discharge

2. That there is unanimity among investigators regarding the expression for the theoretical relation between the speed of the centrifugal pump and the height of lift.

3. That a standard form should be adopted for expressing this theoretical relation.

4. That the most desirable standard form is $\frac{u^2}{g}\left(1 + \text{-----}\right)$

5. That the present formula for the theoretical relation of the speed to the height of lift does not agree with the results obtained experimentally.

Recommendations

With the above conclusions in mind the writer begs leave to make the following suggestions:

The test data on Goulde pumps now in the possession of the Department of Municipal and Sanitary Engineering of the University of Illinois, have not yet been fully studied, and if full details can be obtained regarding the construction of these pumps, it may be possible without further tests to determine either the proper modification of the proposed standard formula for the theoretical lift, or some coefficient for the same, which will harmonize the formula with practical results. The details of these pumps which it is necessary to know, and which at present are not available, are (1) the angle $\underline{\beta}$, (2) the area of the entrance and exit passages of the vanes (involving the width of the runner, the value of $\underline{r}$ and $\underline{r_1}$, and the thickness of the vanes), and (3) the area of the volute or spiral casing at various points, in order to determine the velocity of the water in the casing.

It is recommended that this study be made before any further experiments are attempted, and that similar test data be secured from whatever source possible; it

being borne in mind that the test data are of little value, if unaccompanied by the above-specified data regarding the construction of the pumps. After the formula has thus been studied with the aid of the complete data regarding the construction of the pumps, it will be possible to devise a set of tests, that will be far more valuable than any set which might be recommended as the result of the knowledge derived from the investigation presented in this paper.

It may not be amiss to recommend a careful study of the theories developed by Unwin, and by Hartmann and Knocke. Their method of approach is better calculated to give an idea of the factors affecting the operation of centrifugal pumps than the method of any of the other investigators whose work was examined.

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